Instantons and spontaneous color symmetry breaking

Christof Wetterich

Institut für Theoretische Physik,
Universität Heidelberg,
Philosophenweg 16,
D–69120 Heidelberg, Germany.

Abstract

The instanton interaction in QCD generates an effective potential for scalar quark-antiquark condensates in the color singlet and octet channels. For three light quark flavors the cubic term in this potential induces an octet condensate and "spontaneous breaking" of color in the vacuum. Realistic masses of the ρ- and η'-mesons are compatible with renormalization-group-improved instanton perturbation theory.

It has been argued recently [1, 2] that the physics of confinement in long distance QCD admits an equivalent description in the Higgs picture where color is "spontaneously broken" by an octet quark-antiquark condensate

$$< \bar{\psi}_{Ljb}\psi_{Rai} > -\frac{1}{3} < \bar{\psi}_{Lkb}\psi_{Rak} > \delta_{ij} = \frac{1}{\sqrt{6}}\bar{\xi}(\delta_{ia}\delta_{jb} - \frac{1}{3}\delta_{ij}\delta_{ab}) \qquad (1)$$

The structure in the color indices $i, j, k = 1...3$ and flavor indices $a, b = 1...3$ is such that a physical $SU(3)$ symmetry remains unbroken. The physical vector meson states (gluons) transform as an octet with an equal mass $\sim \bar{\xi}$. They have integer electric charge and can be associated with the $\rho-, K^*-$ and ω-mesons. Also the fermions (quarks) transform as a massive octet (plus a heavy singlet) with the appropriate charges to describe the baryon octet $(p, n, \Lambda, \Sigma, \Xi)$. (We consider here three flavors of light quarks and neglect the

$SU(3)$-splitting in vacuum expectation values due to the mass of the strange quark.) The Higgs mechanism generates a mass for the gluons and therefore provides for an effective infrared cutoff in QCD. This also gives a simple explanation for the confinement of color charges: the gauge fields between such charges are squeezed into flux tubes by effect of the mass, in analogy to the Meissner effect in superconductors. Furthermore, a simple effective action for scalars representing quark-antiquark-bound states $\sim \bar{\psi}\psi$ leads to a very successful phenomenological picture, including realistic pion-nucleon couplings, vector-dominance for the electromagnetic interactions of pions, realistic decay rates of the ρ-mesons into pions and charged leptons and an explanation of the $\Delta I = 1/2$ rule for weak hadronic kaon decays [1].

We propose in this letter a simple dynamical mechanism how spontaneous color breaking is generated in QCD. It is based on the effective 't Hooft interaction for instanton effects [3], [4], in accordance with speculations that instantons are crucial for an understanding of low energy QCD [5, 6]. In short, the instanton-induced axial anomaly induces a "cubic term" in the effective potential for scalar $\bar{\psi}\psi$-states which drives the minimum both for color octet and singlet scalars away from $<\bar{\psi}\psi>=0$.

Besides a dynamical explanation of the color octet condensate our approach also solves two important problems in instanton physics.

(1) First the old question about the effective infrared cutoff for very large size QCD instantons is answered by the colored octet condensate. The induced gluon mass acts as a cutoff, very similar to the W-boson mass for electroweak instantons. Furthermore, the physics of confinement is now integrated in the instanton physics.

(2) Second we solve the problem of the "unboundedness of the naive instanton interaction", which we explain briefly in the following. For three massless flavors the contribution of instantons with size ρ to the effective $U(1)_A$-violating fermion interaction reads* [3], [4]

$$\begin{aligned} d\mathcal{L} &= -d\zeta(\rho)\mathcal{A} \\ \mathcal{A} &= \det\tilde{\varphi}^{(1)} + \det\tilde{\varphi}^{(2)} - \frac{3}{4}(E(\tilde{\varphi}^{(1)}, \tilde{\chi}^{(1)}) + E(\tilde{\varphi}^{(2)}, \tilde{\chi}^{(2)})) \end{aligned} \tag{2}$$

*We take the opportunity to correct the instanton vertex of [2]. It was based on the Fierz transform of an uncorrect vertex quoted in [6].

with quark-antiquark bilinears

$$
\begin{aligned}
\tilde{\varphi}^{(1)}_{ab} &= \bar{\psi}_{L\ ib}\,\psi_{R\ ai} \quad , \quad \tilde{\varphi}^{(2)}_{ab} = -\bar{\psi}_{R\ ib}\,\psi_{L\ ai} \\
\tilde{\chi}^{(1)}_{ij,ab} &= \bar{\psi}_{L\ jb}\,\psi_{R\ ai} - \frac{1}{3}\bar{\psi}_{L\ kb}\,\psi_{R\ ak}\,\delta_{ij} \\
\tilde{\chi}^{(2)}_{ij,ab} &= -\bar{\psi}_{R\ jb}\,\psi_{L\ ai} + \frac{1}{3}\bar{\psi}_{R\ kb}\,\psi_{L\ ak}\,\delta_{ij}
\end{aligned}
\tag{3}
$$

and

$$
E(\tilde{\varphi},\tilde{\chi}) = \frac{1}{6}\epsilon_{a_1a_2a_3}\epsilon_{b_1b_2b_3}\tilde{\varphi}_{a_1b_1}\tilde{\chi}_{ij,a_2b_2}\tilde{\chi}_{ji,a_3b_3} \tag{4}
$$

Partial bosonization[†] replaces the quark-antiquark bilinears (3) by appropriate singlet and octet bosonic fields $\tilde{\varphi}^{(1)}_{ab} \to \sigma_{ab}, \tilde{\varphi}^{(2)}_{ab} \to \sigma^{\dagger}_{ab}, \tilde{\chi}^{(1)}_{ij,ab} \to \xi_{ijab}, \tilde{\chi}^{(2)}_{ijab} \to \xi^{*}_{ji,ba}$. Correspondingly, the interaction (2) transmutes into an effective potential for the scalar fields. An evaluation along the directions (1) and $<\bar{\psi}_{Lib}\psi_{Ria}>=\bar{\sigma}\delta_{ab}$ results for constant ζ in a cubic effective potential

$$
U_{an}(\bar{\sigma},\bar{\xi}) = -\zeta(2\bar{\sigma}^3 + \frac{1}{3}\bar{\sigma}\bar{\xi}^2) \tag{5}
$$

It is obvious that for $\bar{\sigma} > 0$ the effective potential can always be arbitrarily lowered by an increase of the color octet condensate $|\bar{\xi}|$. We will see that eq. (5) with constant ζ is a valid approximation for not too large $|\bar{\xi}|$ and conclude that the instanton interaction induces spontaneous color symmetry breaking. The problem is that the effective instanton potential (5) is unbounded for large $\bar{\sigma}^2, \bar{\xi}^2$. Even though there are, in principle, stabilizing higher-order interactions $\sim \bar{\sigma}^4, \bar{\xi}^4$ induced by anomaly-free loop graphs with eight quark/antiquark legs, it makes no sense that the instanton contribution to the effective action increases without bounds for large values of the chiral condensates.

The approximation of $d\zeta(\rho)$ being independent of $\tilde{\varphi}$ and $\tilde{\chi}$ or, equivalently, $\bar{\sigma}$ and $\bar{\xi}$, holds only for small values of the chiral condensate (e.g. $|\bar{\sigma}\rho^3| \simeq 3/(2\pi^2)$ [4]). The behavior for large values of the condensates is dominated by two effects. (a) For large $\bar{\xi}^2$ the gluons become massive and cut off the instanton contribution. (b) For large $\bar{\sigma}^2$ or $\bar{\xi}^2$ the quarks become heavy and the dependence of the instanton interaction on $\bar{\sigma}$ and $\bar{\xi}$ disappears. We concentrate here on the first aspect. Instead of a careful study

[†]Partial bosonization requires that the effective scalar potential is bounded from below (for details see [2]). We will see that this is indeed the case.

of the dependence of $d\zeta(\rho)$ on the gluon mass and therefore on $\bar{\xi}$ we take a simplified approach which reflects the qualitative behavior correctly: we neglect the influence of $\bar{\xi}$ for small ρ and omit the suppressed contribution for large ρ. As a result, the coefficient ζ depends on the value of the color octet condensate $\bar{\xi}$ by the appearance of an effective cut-off $\rho_{\max}(\bar{\xi})$ in the integral over instanton sizes.

The effective instanton vertex [3] is therefore multiplied by

$$\zeta(\bar{\xi}) = \frac{32}{15}\pi^6 \kappa (f'(0))^3 C_3 \int_0^{\rho_{\max}(\bar{\xi})} \frac{d\rho}{\rho} f(\rho) \tag{6}$$

$$f(\rho) = \rho^5 \left(\frac{\alpha(1/\rho)}{\alpha(\bar{\mu})}\right)^{-\frac{4}{3}} \left(\frac{2\pi}{\alpha(1/\rho)}\right)^6 \exp\left(-\frac{2\pi}{\alpha(1/\rho)}\right) \tag{7}$$

Here $\alpha(\mu) = 4\pi g^2(\mu)$ corresponds to the running coupling in the $\overline{\mathrm{MS}}$-scheme in three-loop order [7] with $\Lambda = \Lambda^{(3)}_{\overline{MS}} = 330$ MeV. The prefactor C_3 reads for the $\overline{MS}$-scheme $C_3 = 1.51 \cdot 10^{-3}$ and $f'(0) = 1.34$. We note that the height of the maximum of $f(\rho)$ depends on the precise definition of α and its β-function. We have therefore introduced in (6) a constant κ of order one which parametrizes this uncertainty. The perturbative value is $\kappa = 1$. Furthermore, κ accounts for the ambiguity in the Fierz transformation of the instanton vertex. We adopt the normalization scale for the fermion operators $\bar{\mu} = 2$ GeV. Due to the strong increase of $\alpha(1/\rho)$ the function $f(\rho)$ vanishes for $\rho \to 1/\Lambda$ and we may take $f(\rho) \equiv 0$ for $\rho > 1/\Lambda$. The maximum of $f(\rho)$ at $\rho^{-1} = 613$ MeV is not very far from the "perturbative range". For the range $\rho^{-1} \geq 800$ MeV the approximation (6) may be considered as a reliable guide, whereas for $\rho^{-1} \leq 500$ MeV it is expected [4] to break down. It seems reasonable to believe the qualitative feature of eqs. (6), (7), namely that $f(\rho)$ suppresses the contribution of very large instantons such that ζ remains finite for $\rho_{\max} \to \infty$. This is, however, not crucial for our argument.

The effective cutoff $\rho_{\max}(\bar{\xi})$ is proportional to the inverse of the $\bar{\xi}$-dependent effective gauge boson mass $\mu_\rho(\bar{\xi})$. By the Higgs mechanism the effective gauge boson mass is, in turn, proportional to the octet condensate $\bar{\xi}$ and the effective gauge coupling $g(\mu_\rho)$

$$\mu_\rho^2(\bar{\xi}) = g^2(\mu_\rho) Z \bar{\xi}^2 \tag{8}$$

We find it convenient to use μ_ρ instead of $\bar{\xi}$ as the independent variable. Inverting the functional dependence $\bar{\xi}(\mu_\rho) = Z^{-1/2}\mu_\rho/g(\mu_\rho)$ one obtains a

lower bound for μ_ρ

$$\mu_\rho(\bar{\xi}=0)=\Lambda \quad , \quad g^2(\bar{\xi}\to 0)\sim \Lambda^2/\bar{\xi}^2 \tag{9}$$

The unknown details of the way how the gluon mass acts as an infrared cutoff are absorbed into a proportionality factor c_ρ of order one

$$\rho_{max}(\bar{\xi})=c_\rho/\mu_\rho \tag{10}$$

Inserting this cutoff in eq. (6), one finds that for $\bar{\xi}\to 0$ the coefficient $\zeta(\bar{\xi})$ becomes almost independent of $\bar{\xi}$ whereas for large $\bar{\xi}$ it decreases rapidly $\sim \bar{\xi}^{-14}$. This qualitative behavior is sufficient for instanton induced color symmetry breaking, independent of the quantitative details. Indeed, the potential vanishes for $\bar{\xi}=0$ and $|\bar{\xi}|\to\infty$ and takes negative values in a range of finite nonzero $\bar{\xi}$. For small values of $|\bar{\xi}|$ and arbitrary nonzero positive $\bar{\sigma}$ the term $\sim -\zeta\bar{\sigma}\bar{\xi}^2$ acts like a negative mass term for $\bar{\xi}$ which destabilizes the line $\bar{\xi}=0$.

On the other hand, for fixed $\bar{\xi}$ and $\bar{\sigma}^2\to\infty$ all effective fermion masses diverge and the instanton contribution depends on the fermion bilinears only through the effective gluon mass or $\rho_{max}(\bar{\xi})$. The potential becomes positive

$$\begin{aligned}&\lim_{\bar{\sigma}^2\to\infty} U_{an}(\bar{\sigma},\bar{\xi})=\\ &C_3\int_0^{\rho_{max}} d\rho\rho^{-5}\left(\frac{2\pi}{\alpha(1/\rho)}\right)^6\exp\left(-\frac{2\pi}{\alpha(1/\rho)}\right)\end{aligned} \tag{11}$$

and this guarantees the boundedness in the $\bar{\sigma}$-direction.

We are now ready to discuss the instanton potential quantitatively by replacing in eq. (5) $\zeta\to\zeta(\bar{\xi})$. In our conventions ζ is positive and the minimum in the $\bar{\sigma}$-direction occurs for $\bar{\sigma}_0\geq 0$. The determination of the ratio of expectation values $r_0=\bar{\sigma}_0/\bar{\xi}_0$ depends on details of the stabilization of the potential in the $\bar{\sigma}$-direction which we do not investigate here. We keep r_0 as a parameter and investigate the potential on the line $\bar{\sigma}=r_0\bar{\xi}$

$$\bar{U}_{an}(\mu_\rho)=-\frac{r_0}{3}(1+6r_0^2)Z^{-\frac{3}{2}}\frac{\mu_\rho^3}{g^3(\mu_\rho)}\zeta(\mu_\rho) \tag{12}$$

The location of the minimum is independent of r_0, Z and the prefactor κ multiplying the integral (6). The vacuum expectation value $\bar{\mu}_\rho$ depends,

however, on the unknown constant c_ρ, as shown in the table (with mass unit GeV). The only scale is set by the perturbative running of α and therefore $\bar{\mu}_\rho \sim \Lambda$.

c_ρ	$\bar{\mu}_\rho$	$\rho^{-1}_{\max}(\bar{\mu}_\rho)$	$\mid <\bar{\psi}\psi> \mid^{1/3}$	ζ_0/κ
1.0	0.65	0.65	0.28	223
1.2	0.74	0.62	0.27	304
1.4	0.84	0.60	0.27	343

Table: Vector-meson mass $\bar{\mu}_\rho$ and singlet chiral condensate $<\bar{\psi}\psi>$.

This establishes our main result, namely that the instanton interaction leads to spontaneous color symmetry breaking, with expectation value $\bar{\xi}_0 = \bar{\xi}(\bar{\mu}_\rho) \neq 0$! The value $\bar{\mu}_\rho$ should be identified with the mass of the ρ and K^* mesons in the limit of a vanishing strange quark mass m_s. In view of the uncertainties the result 600 MeV$\lesssim \bar{\mu}_\rho \lesssim$900 MeV is very satisfactory. It should motivate a more detailed study how the gluon mass term cuts off the instanton integral, which amounts to a computation of c_ρ. (Since fluctuations with $\rho^{-2} < \mu_\rho^2$ are only supressed instead of being completely eliminated we expect $c_\rho \geq 1$). We observe that $\rho_{\max}$ is typically near the maximum of $f(\rho)$ (eq.(7)).

We next turn to the mass of the η'-meson which is directly related to the value of the anomaly potential at the minimum [1]

$$M^2_{\eta'} - \tilde{m}^2_g = -\frac{18}{f^2_\theta} U_{an}(\bar{\sigma}_0, \bar{\xi}_0) = \frac{36\zeta_0}{f^2_\theta}(\bar{\sigma}^3_0 + \frac{1}{6}\bar{\sigma}_0\bar{\xi}^2_0) \tag{13}$$

For f_θ we use [1] $f^2_\theta = f^2(16x+7)/(7(1+x))$ which yields for the average meson decay constant $f = 106$ MeV and large x (i.e. $x = 6$) $f_\theta = 154$ MeV. (The corresponding two-photon decay width of the η', i. e. $\Gamma(\eta' \rightarrow 2\gamma) = \alpha^2_{em} M^3_{\eta'}/(24\pi^3 f^2_\theta) = 2.7$ keV, agrees reasonably well with observation.) We also account here for the effect of nonvanishing quark masses [7] by $\tilde{m}_g = 410\text{MeV}\sqrt{Z_m/Z_p} = 340$ MeV. In the table we show the singlet quark-antiquark condensate $\bar{\sigma}_0 = -\frac{1}{2} <\bar{\psi}\psi>$ ($\bar{\mu} = 2$GeV) which corresponds to $M_{\eta'} = 960$ MeV for $r_0 = 0.5$ and $\kappa = 1$. (Typical values of $\mid <\bar{\psi}\psi> \mid^{1/3}$ for $r_0 = 1/3$ are 10 MeV smaller.) The results agree well with common estimates. We conclude that our estimate of $\zeta_0 = \zeta(\bar{\mu}_\rho)$ leads to a realistic value of $M_{\eta'}$!

We finally point out that the contributions of the nonvanishing strange quark mass to the effective potential are of similar magnitude as the anomaly-induced potential. They tend to increase $\bar{\sigma}_0$. It is interesting to note that the anomaly in two-flavor QCD has also the tendency to induce an octet condensate due to a negative quadratic term for $\bar{\xi}$.

In presence of a (not too large) strange quark mass and the four fermion interactions generated by QCD-box diagrams the effective potential acquires additional contributions[2]

$$\Delta U(\bar{\sigma}, \mu_\rho) = -2m_s\bar{\sigma} - \zeta^{(s)}(\mu_\rho)m_s(2\bar{\sigma}^2 + \frac{1}{6}\bar{\xi}^2(\mu_\rho))$$
$$+\frac{g^4(\mu_\rho)}{16\pi^2\mu_\rho^2}\left(\frac{g^2(\mu_\rho)}{g^2(\bar{\mu})}\right)^{-\frac{8}{9}}(3L_\sigma\bar{\sigma}^2 + \frac{4}{3}L_\chi\bar{\xi}^2(\mu_\rho)) \qquad (14)$$

The gluon mass acts as cutoff in the box diagrams[‡] and we neglect the effect of the fermion mass, resulting in $L_\sigma = \frac{552}{117}L_\chi = \frac{23}{9}$. We note $d\zeta^{(s)} = (5/6)(\pi\rho)^{-2}(\alpha(1/\rho)/\alpha(\bar{\mu}))^{8/9}d\zeta$ and an additional contribution to the η'-mass $\Delta M^2_{\eta'} = 16m_s\zeta^{(s)}(\bar{\mu}_\rho)\cdot(\bar{\sigma}_0^2 + \bar{\xi}_0^2/12)/f_\theta^2$.

In conclusion, the instanton-induced anomalous six-quark interactions induce the spontaneous breaking of color. The destabilization of the "color-symmetric state" $\bar{\xi} = 0$ due to the instanton interaction seems to be quite robust. We could not find competing stabilizing terms from other effective interactions in QCD. The pure instanton interaction with fluctuations evaluated perturbatively and quark mass effects neglected gives already a very satisfactory picture with a realistic range for the vector-meson masses and the mass of the η'-meson.

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